Making Math Meaningful

INCREASING STUDENT ENGAGEMENT IN MATH THROUGH AN

IMAGINATIVE EDUCATION APPROACH

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2 Real World Math Problems (in the classroom)

PROBLEM 1

Can I ask you to please recall a time when you really loved something; a movie, an album, a song or a book. And you recommended it whole heartedly to someone you also really liked. And you anticipate that reaction, you waited for it, and it came back, and the person hated it. By way of introduction, that is the exact same state in which I spent every working day for the last 6 years. I teach Highschool Math.

(Dan Meyers, Ted Talk, 2010)

As I heard Dan Meyers relate his experience of teaching math to students who hate math, I feel a sense of comradery with this man I have never met. While I do not teach just one subject – math – it is the subject that I most value. I have always loved teaching math, and perhaps my infatuation with the subject blinded me to how many of my students felt towards this subject.

PROBLEM 2

In the past couple of years, I have often found myself telling my students that I do not want them to think differently when they walk into the math classroom – I want them to use their common sense. I can remember giving a math assessment to my Grade 7/8 math class last year, where they had to make up a problem that used proportional reasoning with decimal numbers. One student created a problem that started “You are having a party with 10 friends and you have an 8.5L bottle of Coke.” The image in my head of a massive, impossible to pour, water-cooler-sized Coke bottle had me shaking my head. On the same assessment, another student wrote “You and your class are sharing 4 cookies. How much of the cookie does each person get?” Either these were dinner platter-sized cookies, or we would need to smash these cookies into smithereens to be able to distribute them evenly amongst our 30 students. The lack of common-sense thinking shown in these examples told me that something was not quite right in my math class.
PURPOSE

This school year, I feel my brow furrowing deeper and deeper down my face as I encounter the math buzzwords: *rich mathematical tasks, open questions, low-floor high-ceiling, 3-Act Tasks, manipulatives, number talks, student collaboration, et cetera.*... My brown furrows even further when I think about my students this year and their seeming lack of interest in math, and lack of ability to apply math concepts in day-to-day situations. I take the lack of engagement personally: there is an implication that I am doing a disservice to students in the way that I currently teach math when my students say things like “I hate math” or “math is trash”. The reflective teacher in me, and my propensity towards obsessive compulsions, causes me to over-think my ability as a teacher, and spiral into anxiousness with feelings of being inadequate at my job, and I fixate on how I should change my ways.

As a generalist teacher (meaning I teach all subjects), I do not often take the opportunity to delve deeply into learning about the different strategies and pedagogies around teaching the various subjects. My preparation time is spent preparing lessons and assessments for all the subjects that I teach whilst trying to understand BC’s Revised Curriculum, how to assess Curricular Competencies rather than content knowledge, and how I will infuse reflection of the Core Competencies into my students’ days. Aside from the occasional Professional Development opportunity, I feel that there is not enough time allotted for teachers to engage in meaningful research about their practice. And so, a master’s program affords me the opportunity to delve deeper into understanding my pedagogies, their theoretical background, and ultimately, figuring out what kind of teacher I want to be.
I chose the Imaginative Education program, as I felt I learn how to make education more engaging for my students. The word *imaginative* conjures the ideas of kids being so engrossed in the school day that they never look at the clock to see how long it is until lunch; or that they don’t ask to go to the bathroom, to get a drink, all to escape the monotonousness of their school day. My goal as an educator is to help students enjoy the act of learning about their world. Unless the kids are going to the bathroom to see how the mechanics of a toilet allow gravity to pull the water down or how getting a drink of water affects their bodies, I see their need for a break as a sign of task overload or their boredom with the tasks we are working on.

This action research opportunity has allowed me the opportunity to delve into one of my favourite subjects – math. Funnily enough, I recently learned that it is the majority of my students’ least favourite subjects (cue the spiraling on my inadequacy as a teacher). My goal in this project is to see if changing my approach to teaching math has any effect on my students’ dispositions towards the subject. My previous approach to teaching math was as follows: teacher-led lessons while students take notes; students practice concept with textbook questions; teacher gives assessment using alternative textbook questions; start next unit.

**ACTION RESEARCH PROPOSAL**

Kieran Egan’s Imaginative Education discourse provides an approach to teaching that suggests that by using cognitive tools to engage learners’ imaginations, teachers can affect student engagement with the topics we teach (Egan, 1997; Egan & Judson, 2016). Up for the challenge, I have decided to investigate the effects of an Imaginative Education approach to teaching math, specifically looking to see if I can increase engagement with rich math tasks, foster a deeper
understanding of math concepts, and to help students recognize their own numeracy skills. Through an Imaginative Education framework and with the use of rich math tasks, I will teach a Ratios unit and see if this approach affects student attitudes and behaviours in math.

Rich math tasks can be described using Nancy Butler Wolf’s 6 characteristics of these types of tasks. Rich math tasks are:

- Accessible to all learners ("low floor, high ceiling")
- Real-life task or application
- Multiple approaches and representations
- Collaboration and discussion
- Engagement, curiosity, and creativity
- Making connections within and/or across topics and domains [Boaler, "Mathematical Mindsets"]
- Opportunities for extension

(Audet, 2016)

I can see that these characteristics fit nicely with Kieran Egan’s Imaginative Education framework. Many educators say that approaches such as 3-Act Tasks, open-ended questions, and parallel tasks, and other rich math tasks are great tools to make math more meaningful for students, and being that Imaginative Education is an approach that makes the curriculum more meaningful, I suggest that rich math tasks are effective because they employ the cognitive tools required to make learning meaningful.

And so, my question (framed as a fraction where the numerator represents the “parts” of math education and the denominator represents their “shared trait”).
OTHER GUIDING QUESTIONS

➢ How can I increase the number of students who participate in math?
➢ Is engagement/participation the most important factor in learning?
➢ What is the research that supports the current or “new” approaches to teaching math?

Situating Myself

MY EDUCATION

My education has profoundly shaped my pedagogy and my beliefs about the purpose of education. Prior to even entering school in the early 1980’s, my mom trained me with flash cards, having me memorize math facts, how to spell words like ‘giraffe’ and ‘medicine’, and the names of complex shapes. I went into Kindergarten ready to memorize all the facts. I could not wait to do homework, sit at a big-kid desk (you know, the one with the bar connecting the seat to the table), and I was confident that I would be good at it all. Lucky for me, my education at public school in British Columbia between 1985 and 1998 was filled with tests of memorized facts, spelling, and proper grammar. I finished high school having achieved A’s and B’s in every subject of every term, qualifying me for the special golden-colored honour roll cord. I felt smart.

Not only was I “smart”, I was also good at the routines of school. I never skipped classes, was polite to my teachers, always did my homework, and I followed all the rules. I had all my teachers’
approval. I knew that I was becoming a responsible, well-adjusted adult that would fit nicely into society.

**MY CONCEPTUAL FRAMEWORK**

My ideas about the purpose of education are grounded in Durkheim’s ideas of socialization of the child (Durkheim in Hoenisch, 2005). As Kieran Egan says about socialization, “the central task of socialization is to inculcate a restricted set of norms and beliefs – the set that constitutes the adult society the child will grow into” (Egan, 1997, p. 11). I exited high school and entered “the real world” with a sophisticated understanding of how to fit-in. I followed society’s rules well, never broke the law, and felt tremendous shame and guilt the one time I slept in for my 6am White Spot shift. I grew up, got my degree, got married 6-days later to a man, bought a house in the burbs, got my contract as a teacher, had two kids, and now it is time to further my education. I am fitting nicely into the expected social norms of our western society, aren’t I?

I entered teaching with the strong belief that it was my job to shape students to be productive, rule-abiding citizens, much like I had been. I put tremendous importance not so much on achievement, but rather on the skills of a responsible citizen; on arriving to class on time, being prepared with necessary learning materials, completing work on time, attending classes regularly, politeness to one another (especially towards your elders), an ability to work cooperatively, and a good ability to hold your bodily functions until break times. Not only do I lay out these expectations quite clearly each year, I also expect students to hold these as values for themselves. Except for very few students, I assumed that most of my students also believed that they were at school to learn how to be responsible citizens, and that they valued these skills as much as I did.
Egan discusses how our presuppositions shape our curricular and pedagogical decisions. My need and desire to use education to shape kids into contributing members of society is rooted in my presupposition that human nature is inherently bad, and that left to their own devices, students will not adhere to the social norms (Egan, 1999). I illustrated how this conception has shaped some of my pedagogical decisions, but there are curricular implications to this presupposition as well.

I believe that students learn math so that they can contribute to society by paying the appropriate amount of taxes, budget to buy a car or home, and to avoid being taken advantage of by non-law-abiding citizens. Skills like calculating percent, comparing ratios, and reasoning proportionally were what I valued most in math, and this value fit well with my idea of the purpose of education. With these skills came other subsets of skills such as being able to add, subtract, multiply and divide whole numbers, integers, and fractional numbers. Much of my math program (up to now) was based in having kids practicing these skills, then testing them on their ability to memorize the procedures required to solve problems involving these concepts. I was convinced that students performing well on tests equated to future citizens who would pay tax, budget appropriately, spend wisely.

My own math education was mostly based in being taught “the way”. Memorizing math facts and procedures was how I succeeded in math, and I did not know that there could be another approach. I am starting to recognize, however, that procedures in math are like grammar in English – they are important to understand to be able to engage with the topic, but they are not the engaging part of the topic. While I briefly touch on grammar when we are writing comparative essays, much of the lesson on writing an effective essay is based on creating an effective argument. When students are reading, they need to know rules about phonics, and exceptions to these rules,
but they are not what makes the story engaging. Likewise, in math, knowledge of algorithms is necessary to calculate numbers, but it is a means to an end. In the end, I want students to appreciate the way that math allows us to understand the world through patterns and relationships, how it can help them be savvy shoppers, good home renovators, and effective time managers. This understanding of the topic requires more than just memorizing a set of algorithms.

As this school year has progressed, I have felt a sense of disinterest from my students in math. During lessons, I noticed that my students were staring off, not taking notes, asking for frequent bathroom breaks, and when it came to the practice of math, there was a lack of perseverance with solving word problems, and if attempted, students wanted to know “the way”.

Perhaps, in all the 14 years I have been teaching, my students have always shown this disinterest, but I have been unaware. My journey through the Imaginative Education Master’s program has taught me the importance of engaging the students’ emotions in learning, and now I am very much in tune with student engagement (or lack of engagement) with the activities we do in class. Throughout this action research, I am not only attempting to make math more meaningful for students, I am attempting to change my beliefs about the best practices when it comes to math – no easy feat for a seasoned teacher.

What Say the Experts?

LITERATURE REVIEW

On my search for a deeper understanding of what was happening in my classroom, and how I might affect change, I investigated what good math programs look like, and how some changes to my teaching techniques could help enhance the meaningfulness of the topic of math. I started
researching some of those ‘buzzwords’ I noted at the beginning of this paper and found three themes common among the literature on making math meaningful and engaging for students: evoking a sense of relevance, creating a sense of play with the topic, and establishing a sense of competence. I situate these themes in the Imaginative Education Framework, showing a common understanding of how best to teach students. But first, I wanted to understand some of the research on engagement in the math class.

Engagement is a complex topic that is multifaceted and widely defined, and thus difficult to measure and interpret. Yet, it is an important indicator of academic achievement and life-long learning (Fredericks, 2004). The research that resonated with me the most was by Jennifer Fredricks, a Connecticut College professor of Human Development. She recognizes the challenges of coming up with ways of increasing engagement (or decreasing disengagement) and suggests “more multi-method, observational, and ethnographic studies would contribute to this effort” (Fredericks et al, 2004, p. 87). Her qualitative research on developing a survey of math and science engagement uses multiple methods of data collection, analysis and multiple member checks; draws information from a diverse population of students and teachers; and grounds its findings in previous research done on engagement. Ultimately, Fredericks and her team came up with what I deem to be a strong student survey of engagement in math and science. This survey measures behavioral, emotional, cognitive, and social engagement of students in the math and science settings. While this survey does not give information about the factors that have impacted student engagement, it gives students an opportunity to identify their own engagement in a math classroom. I used this list of survey questions to develop my own qualitative survey (Appendix A) that had my students write about their behaviors, emotions, cognitive processes, and social aspects of their learning in my math class.
The results of this survey showed that they found math to be irrelevant to their lives now, boring, or too hard, which fits nicely into the themes I mentioned at the beginning of this section.

**EVOKING A SENSE OF RELEVANCE**

*To do well in math class, children know that they have to suspend reality and accept the ridiculous problems they are given. They know that if they think about the problems, and use what they understand from life, then they will fail. Over time, schoolchildren realize that when you enter Mathland, you leave your common sense at the door.* (Jo Boaler, 2015, p. 51)

In the quote above, Jo Boaler describes the traditional math class as one that is separated from reality. Her discussion around math being a make-believe, made-up reality resonated with me as I reflected on the 8.5L bottle of Coke in a former student’s math problem. Boaler recognizes the issues in the way math is taught today in that it does not reflect the true nature of what math is about.

*When we look at mathematics in the world and the mathematics used by mathematicians, we see a creative, visual, connected and living subject. Yet school students often see mathematics as a dead subject – hundreds of methods and procedures to memorize that they will never use, hundreds of answers to questions that they have never asked.* (Boaler, 2016, p.31)

She goes on to criticize the “pseudo contexts” that are often found in our textbooks. Problems like “trains speeding towards each other on the same track” do not represent reality (Boaler, 2016, p. 194). One might call this train problem a good use of the imagination, but as Wordsworth has said, “‘imagination is reason in her most exalted mood’” (in Egan, 1997, p. 56). Imagination is not made-up content, rather it is a means of making sense of the world.
In a qualitative longitudinal study, Catherine Attard was able to discern some characteristics of math lessons that students found to be engaging (2011). After her 3-year study on 20 middle school students in Australia, she found that “the incorporation of tasks that mirrored life-like situations appears to have been a strong factor in engaging students in the mathematics tasks, as were the tasks that required the students to take the mathematics out of the classroom and into the school playground” (Attard, 2011, p. 372). While this one study reveals what Jo Boaler has also found, it did have some limiting factors to its breadth. Attard selected 20, high-achieving, Catholic school students to follow and interview over the course of 3 years. Because she was looking at factors of positive engagement, she felt it was necessary to use students who positively engaged with mathematics. I would like to see further studies done on more diverse populations of students to see parallel findings. Despite the limitations of this study, it adds to the suggestion that for math to be engaging for students, students must be doing “real” tasks, instead of the ones commonly found in “Mathland”.

I see a strong connection between the Imaginative approach to education (Egan, 1997), and the studies around meaningful math tasks. As my students are middle school aged, I look to what Vygotsky has said about middle school students’ imaginations. As Natalia Gajdamaschko notes about Vygotsky’s work on middle school students:

...imagination undergoes a revolutionary shift, a shift that profoundly impacts students’ intellectual development, personality, behavior, and ways of understanding and making sense of the world. According to Vygotsky, it is in this imaginary world of imaginary heroes, testing of boundaries, and imaginary intellectual games that the real battle for the development of personality, identity formation, and development of thinking is fought out in school years. (as cited in Gajdamaschko, 2006, p. 14)
Kieran Egan describes the intellectual processes of students at this age as Romantic Understanding. He says that “…attending to the characteristics of Romantic understanding will provide the most effective means of ensuring that students master whatever knowledge and skills they need in order to deal successfully with the world” (1997, p. 94). Egan describes the Romantic understanding coming from a time in our cultural history when there was a desire to both evoke emotion and represent reality as it really was (Egan, 1997). Kieran Egan and Gillian Judson note that, “Children from age 7 or 8 to around 15 do not so easily accept the existence of fairies, the Eater Bunny, or magical kingdoms accessible by beanstalks. Instead they become increasingly interested in making sense of the real world” (Egan and Judson, 2016, p.70). I equate math problems involving trains speeding at one another on the same track or 8.5 litre bottles of Coke to things of fantasy (or to quote Jo Boaler, “Mathland” tasks) and recognize the importance of using real world math contexts do a better job of engaging the romantic cognitive tools available to my middle-school students.

Another aspect of the Romantic kind of understanding involves acknowledging that children at this age are beginning to recognize that they are becoming a part of a society that has rules and constraints. As a result, students begin to identify with qualities of characters that “…transcend the constraints and triumph over those who constrained them, whether parents, teachers, conventional behavior, or adult society in general” (Egan, 1999, p. 47). In theory, by finding the transcendent quality of a topic in math, students in my middle school class will emotionally engage with its content. By framing my ratios math unit around the idea that students will become good at arguing for what is fair or unfair, not only do I engage with the ways they are beginning to now make sense of the world, I am also teaching them how to form a good mathematical argument in real contexts. Which middle school student does not want to be able to argue effectively with mathematical reason about the constraints and rules that they find unfair? For example, if a student is 1 minute
late for class, should their consequence be equivalent to the student that was 15 minutes late for class? Or if 3 students are chosen to do a job and paid with sour keys (the current currency in the middle school classroom), and one of these students do not do a fair share of the work, what is a fair distribution of sour keys? This kind of approach to framing a unit of study not only shows students the relevance and benefits of having this kind of knowledge in a real-world context, but it also engages the emotions of a middle school student by revealing how the transcendental quality of fairness in proportional reasoning can help them fight against some of the rules they find constraining.

CREATING A SENSE OF PLAY

Another theme that came up in the review of the literature is the need for students to see math as a playful and creative endeavor. I chose the word ‘play’, as I felt it best represented the three ideas I was trying to capture. In the context of the math class, I see play in three ways: the act of playing with ideas; as playing with manipulatives and visuals; and playing through cooperative endeavours. Much of the research on math instruction has shown that students quite often see math as a series of processes and rules that need to be followed, when, in fact, engaging in math involves creative, visual, and collaborative processes when solving problems. (Boaler, 2015; Shepard in Egan and Nadaner, 1988; Attard, 2011; Liljedahl, 2005). Marian Small notes that compared to the math education that most of us grew up with, “Students are more likely to use manipulatives and technology than in the past, teachers are more likely to encourage students to use personal strategies, and there is typically much more discussion in the classroom” (2010, p. 29). She recommends that teachers use “Open Questions” (questions that have multiple answers) to encourage students to think of their own ways of working through a problem and to promote
conversations about mathematical ideas. These kinds of approaches in teaching math would have students start to see the importance of their own thinking and ideas, as well as the importance of collaboration and creating ideas together.

Jo Boaler suggests that when students are asked to visualize and draw their thinking, there are higher levels of engagement (2016). She also discusses the importance of students seeing the patterns in math: “At its core, mathematics is about patterns” (2016, p. 23). For students to really understand math, they need to be able to see the patterns and recreate them for future applications. Boaler suggests using manipulatives to help students see some of the visual components in math (2016). If teachers are teaching math in a way that encourages ‘play’ (play with their ideas, play with manipulatives, and play together), one would suppose we will see more engagement in the math class.

The ideas of making math a more playful endeavor for students can be situated in the Imaginative Education theory. While above I suggested that my middle school students are most likely working with cognitive tools in the Romantic Understanding, some of the ideas above fit into both Mythic and Romantic kinds of understanding. Egan suggests that it is important to use as many cognitive tools in the former kinds of understanding to “minimize the losses” that come with our literacy gains (1997, p. 7). Using games and open-ended questions, draws on a cognitive tool from the Mythic kind of understanding. On the Imaginative Education website, it is noted that incorporating play in the classroom gives students the opportunity to “…make certain decisions, assess the outcomes of their decisions and, possibly, deal with the consequences of their decisions” (www.ierg.ca, 2018). By using Small’s “Open Questions” strategy and encouraging discussions amongst their peers, students are engaging in play with their ideas. Math games are another way to
incorporate play, where students must make decisions about best deals or potential outcomes to do with probability. Students “playing” with their ideas should increase engagement and learning in the math classroom.

The use of collaboration and having students be the owners of the knowledge is drawing on the cognitive tool Egan refers to as Change of Context. He and Judson note that “One of the enemies of effective teaching and learning is students’ and teachers’ boredom. One of the triggers of boredom is excessive familiarity and taking things for granted” (2016, p. 111). By setting the lesson up so that students work together to share ideas and build off one another’s ideas is both a playful process and one that changes the contexts of many traditional math classrooms. A traditional math classroom would have the teacher and the textbook as the source of a math lesson. Changing this context by allowing students to depend on their previous knowledge, and the knowledge of their peers, not only provides a little ‘shake-up’ to the often monotony of a math class, it also shows students that they are the meaning-makers – that their knowledge is a product of their living tissue (Egan, 1997).

Play in math draws on another cognitive tool from the Mythic kind of understanding: the use of patterns in numbers. Patterns are a useful tool to aid in memorization, which was especially useful in a time when we did not have written language (Egan, 1997). Hence, it is tool that can help us to remember certain aspects of numbers. For example, if we think about the multiples of 5, we notice that every second number ends in zero, whereas the alternate numbers end in 5. This pattern helps students to remember their 5-times tables. These patterns can also be revealed visually, such as on a one hundred grid. Highlighting the multiples of numbers shows visual patterns that can helps students remember certain math rules (example adapted from www.ierg.ca). These visual patterns
end up creating graphic organizers, a cognitive tool drawn from the Romanic kind of understanding. “Visual tools such as lists, flowcharts, and diagrams will make it easier for the eye to retrieve information” (Egan and Judson, 2016, p. 122). Drawing from what Boaler claims about math being all about patterns, we can easily see the necessity of having students identify and play with these in the context of a math class. By having students play with these patterns, and find patterns of their own, we engage Mythic and Romantic cognitive tools allowing for more engagement and thus memory of math concepts.

**FOSTERING A SENSE OF COMPETENCY**

Much of the research I have done on the disengagement students feel in math has to do with their self-efficacy in the subject (Boaler, 2016; Small, 2012; 2018). Both Jo Boaler and Marian Small write and talk about how traditional approaches in teaching math can lead to math anxiety in students. Traditional approaches refer to teaching math as a set of rules, valuing speed, and the emphasis on “right and wrong” (Small, 2012). Jo Boaler says, “Too often, [math] is taught as a performance subject, the role of which, for many, is to separate students into those with the math gene and those without” (2016, p. 93). Boaler and Small advocate for “equity” in math by providing “open-ended tasks” (Boaler, 2016; Small 2012) that give all students an entry point to the task at hand.

One approach to teaching with open-ended tasks (also referred to as “Low-Floor, High Ceiling Tasks”) is with the 3-Act Task, a method developed by math educator Dan Meyer. This approach uses video and visuals to show a real-world situation that can use math concepts to solve. Meyer discusses how this story-approach to teaching a math concept allows all learners to engage with the topic. “Act One attempts to lower barriers to entry. It’s visual. It requires very little literacy
from the student. It’s perplexing” (Meyer, Teaching with Three-Act Tasks: Act One, 2013). The first act is like the “hook” from a story. It presents a mathematical situation without numbers, and asks students to record what they see, and what they wonder. After this process, students agree on a mathematical wondering they will solve. At this point, students are grouped in 2 or 3, and work together to solve the question. These types of tasks are said to teach students “how to justify thinking and answers (rather than relying on a teacher or answer key for validation of correctness)” (Richard Woods, n.d.). By using visuals and videos, and asking students to observe and wonder, all students able to engage with the beginning of this kind of math task, without the fear and anxiety of what skills they may or may not have to solve the problem.

While an imaginative approach to education uses cognitive tools to engage the emotions of students, Egan does not address how this kind of engagement may enhance a sense of competence within a student. As noted in the literature on the topic of engagement, student engagement is linked to success with subjects (Fredericks, 2016; Attard, 2011). By changing the context of a math class, and presenting a perplexing story that students need to resolve (as in a 3-Act Task), all students can engage with the task without feeling the anxiety of having to know certain math skills. Most students in any given class can observe videos and wonder about the situation, thus “hooking” the student into the learning that is about to happen. A traditional math class, however, often starts a lesson with assumptions of previously learned content. For example, a traditional Grade 6 Equivalent Fractions lesson starts with the assumption that students understand fraction representations, divisibility rules, and factoring skills. It then goes through a series of algorithms that students are expected to then practice and apply. I would argue that by changing the context to present math in these open-ended ways can both be interesting (as opposed to boring) as well as help to foster a sense of competence in students.
3-Act Math Tasks and other open-ended tasks employ another cognitive tool – Humanizing Meaning. Egan and Judson talk about the importance of revealing knowledge as a product of “human hopes and fears” and that knowledge is not in books. “…knowledge is a function of certain living tissue in our bodies, and in our minds it looks nothing like the neat lines of text and numbers in the books” (2016, p. 94). Egan would suggest that to find the human meaning in topics, we need to look to the original thinkers of the knowledge (in math, we would use Pythagoras, Archimedes, Newton, and many other great thinkers of the past). However, another way to frame “humanizing meaning” is to allow students to use their own previous knowledge to work with math concepts. In Peter Liljedahl’s work on fostering the “Aha!” moments in math, he says “…the humanizing of mathematics need not be centered in other humans' involvement of mathematics and mathematical activity, but in the individual student's own involvement in mathematics” (1993, p. 207). Open-ended tasks allow for students to think about mathematical situations using their own knowledge, rather than thinking they must rely on a set of math skills or competencies to work with the task. This legitimization of students’ previous knowledge and skills will help to foster that sense of competency that the research has proven to be necessary in learning.

Research Site

THE COMMUNITY

I have been teaching at my current school since it opened in September 2013. This middle school is in a wealthy community. Many of the students of in my school have laptops, smart phones, and go on trips abroad (as I type this, 4 of my students are on international trips for Spring Break). We are a BYOD school (which stands for Bring Your Own Device) and 22 of 25 of my students bring their own laptops from home, while the other 3 borrow laptops from the school.
In the first year we opened, our staff began using the BC Revised Curriculum, even though it was still in the draft form. We eliminated the individual subjects of Social Studies, Science, and Language Arts and called it Integrated Studies. We also decided to do away with letter grades on Report Cards, using the Ministry Proficiency Language, and comments. These changes sparked a lot of questions, and even a parent-started Facebook Page that criticized our efforts and threatened a lawsuit to bring back the letter grades. These somewhat controversial moves not only sparked questions from our parent community, it attracted media attention and our principal at the time was interviewed by the CBC as well as numerous newspapers. Now, our school has a reputation for being student-centered, progressive and near-impossible to get into, unless you live within the catchment prior to the school year starting.

THE CLASS

For the past 4 years at this middle school, I have taught mostly Grades 7 and 8. This year, I teach almost exclusively Grades 6 and 7. My classroom shares an opening wall with another Grade 6/7 classroom. This is the first year that my teaching partner and I have opened the wall to co-teach math. Between the two classes, there are 51 students of which 19 are Grade 7 and 32 are Grade 6. In this make-up there are 3 students who have been identified as Gifted, 3 English Language Learners, 4 students with Learning Disabilities, and 1 student who is hard of hearing. We also have several students who are highly anxious, and 2 students with ADHD. Amongst this group, there are 2 students who often do not do any work in class, and often scoff at the activities we do in class. This year, both my teaching partner, Andrea, and I teach 4 days per week, leaving us with a third teaching member, Alice, who teaches in Andrea’s class on Wednesdays and in my class on Thursdays. We seat our students in groups ranging in sizes of 3 to 8, and they are an energetic group
of students. Students have the flexibility to move in and out of the classroom to alternative locations to work, and it is rarely quiet in our classes. For the most part, our students love any activity that involves movement, food, and a change of environment.

**Research Methods**

The math unit on Ratios and Percent began on January 29th and ended on March 11th. During this time, the bulk of my data collection took place from January 29th until February 19th, which was the Ratios portion of the unit. When I finished the Ratios unit, I continued to use a similar approach to teach a unit on Percent. I taught my math unit over an average of 3 days per week for a 70 – 90-minute period. I was present in the math class as both teacher and observer for 15 days during this time frame, however the students had math on 20 of these days. I do not work on Thursdays, leaving my two teaching partners to teach the math lessons that I prepare. In addition to this, three of the weeks had multiple days of interruptions to our normally scheduled math instruction including a previously arranged Skype with an author (February 4) a Snow Day (February 12), a School Closure Day (February 15), Family Day (February 18) and a Professional Development Day (February 22).

These “senses” or feelings that I hoped to cultivate through my unit can best be assessed through qualitative means, however I also use some quantitative analysis to look at my data. I developed a unit for the topic of Ratios that employed rich mathematical tasks (3-Act Tasks, open-ended questions) and used some of the cognitive tools of the Mythic and Romantic understanding. I added to my preliminary data with a survey on attitudes and beliefs about math prior to starting my unit. Throughout the math unit, I collected observations and student anecdotes during the rich math
activities, observations and student comments during class conversations, results of a student assessment and post-assessment reflection, and, to corroborate my findings, I had several conversations with my teaching partners. Ultimately, I was looking for changes in student engagement with math.

INITIAL OBSERVATIONS OF MY MATH CLASS

As I mentioned previously, I have noticed a lack of engagement in the math lessons I was leading. While I used to blame this on the fact that the students were not as mature as my previous Grade 8 students, and that kids just don’t have the same attentions spans that they used to, I am recognizing that my approach to teaching has some flaws. The following conversation took place at the beginning of January this year:

Teacher observes student during a lesson led by another teacher and notices his back towards her, not taking notes, staring off at the window.

Later, in conversation with student...

Me: You had your back to the teacher while she was talking during the whole lesson.

Student: Yeah, but we weren’t doing anything.

Me: Yes, we were talking about Mean, Median, Mode and Range. We were discussing the Median Canadian Salaries compared to the Mean Canadian Salaries.

Student: (with look of disdain) Yeah, but there was no work to do.

This vignette is one of many examples that illustrate the lack of participation in the math lessons that both my teaching partner and I had run over the previous 5 months. While I had recognized this disengagement in the past, my Action Research project topic forced me to think more in depth
about this interaction with my student. I jotted it down in my brand new, leather-bound AR journal immediately, and paused to reflect on the meaning of his words.

I got stuck on the word “doing”. This student was not “doing” anything, because there was nothing to “do”. Not even the act of making meaning. Thinking back to the lesson, how many 11 and 12-year-old students really care about the average Canadian Salary? While this was a Real-World scenario using mean, median, mode and range, it did not have that relevance or play that the experts have deemed necessary for engagement. The result? This student stayed in at lunch to re-learn mean, median, mode, and range because he did not understand it, despite using what we thought were “relevant” tasks.

Further to my own observations, my teaching partner has noted on several occasions that while I am teaching a math lesson, she is having to wander around the room to remind many students that they need to be copying the notes from the board in their notebooks. These were also the same students who required additional assistance when it came to ‘doing’ the practice questions from the textbook.

**INITIAL SURVEY**

When I originally crafted the initial survey (Appendix A) that I gave to my students, I thought that I would be able to use their responses to find out if there were one or more areas that students felt engaged or disengaged in math (cognitive, behavioural, emotional, or social). I thought that I would be able to cross reference their responses with my categorical list containing the types of engagement that corresponded to each question and be able to craft myself a neat little circle graph
identifying the kind of engagement they most felt in math. Instead, I discovered some overwhelming themes that emerged from their responses. Overall, my findings were that students felt math was boring, for future use only, and too hard.

In response to Question 1, *Why do you think we learn math at school*, and Question 2, *Is math important? Why or why not?* 39 out of 44 respondents wrote a statement that alluded to math being relevant for when they are older and for their jobs. Some sample written statements include:

- “We learn math to prepare us for the future.”
- “I think we learn math at school because so when we are older we know math so like when we buy something we can do quick math to buy something. When you get a job you need to know this. Even if your not doing any of this you still need to know math.”
- “So when we go out into the real world we are prepared and don’t end up living in a box on the side of the road.”
- “So we know it when we get jobs.”
- “To better prepare for life in the future. There is a need for mathematical competence in almost every job available today, so we start honing our ability from a young age, which makes your math stronger and better as you grow up.”

Looking at the responses from the five students who did not say that math was only important for their future, 3 students answered without specific application of math:

- “Because it applies to everything in life.”
- “We learn math at school, so we can learn new skills and get better.”
- “We do math because in life if you come to something difficult that math could help you with, knowing math would make it a lot faster and easier.”

The two students that gave specific applications of math mentioned baking, shopping, and being trapped and needing to decode numbers to get out.

Overwhelmingly, the response to this question identified getting jobs, paying for things (like taxes and parties), and being able to survive in the real world as the main reasons we learn math at school. From these responses, I could tell that most of our students did not feel a sense of current relevance for learning math. Once I made this discovery, I could start to understand why I have
noticed a lack of engagement in my math classroom. If they truly feel that math is only important for their future, then what would be the incentive to learn it now? Can they not just “Google it” when they need it later? Upon this realization, I felt a sense of urgency to start finding activities that made math meaningful to Grade 6 and 7 students.

In response to Question 7, *Do you participate in the math conversations we have during lessons we do in class*, students answered yes or no, then selected any number of reasons to explain why they did not participate. Out of the 44 students who did the survey, four did not respond to this question, and 11 students responded that they usually participated in the conversation. 33 students responded saying that they did not participate, and the following chart shows the frequency of reasons selected.
Of the students who answered “other”, three of them wrote that they would be embarrassed if they got the answer incorrect, one said they preferred to watch and listen, one said they did not want to, and one said they sometimes get too tired.

The most interesting part of these results were the names of some of the students who selected “I am unsure if I have the right answer”. Some of these students are who I would describe as confident, competent math students who always perform well on assignments and assessments. My analysis of these responses showed me that my students lacked confidence in their abilities, but also made me start to question the efficacy of having 51 students in one room for stand and deliver math instruction and teacher-led conversations. I can remember being a shy student in Grade 7, not wanting to share my answers in math in front of my 28 peers for fear I might be wrong. I can only imagine that the number of students in the class is inversely proportionate to the number of brave students willing to risk sharing a wrong answer. This answer convinced me that the teacher-led, stand and deliver way I taught math needed to change to get more students participating in the math conversations. This led me to plan math to include smaller group conversations.

The last question that struck me on this survey was Question 11 Do you think most people enjoy math? Why do you think this? 40 of 44 students responded to this question, with 4 students saying ‘yes’, 11 students ‘saying some people do, some people don’t’, and 25 students responded saying that ‘most students do not like math’. The most common reasons given for not enjoying the subject was because it was too hard or too boring.

From these three survey responses, I was beginning to understand why my teaching partners and I had been observing a noticeable lack of engagement during math class. First, the survey data showed that many students did not see the relevance of the topic in their current lives. Second, it
appeared from the survey data that many students did not have the confidence to participate in the math conversations during our large class discussions. Third, many students felt that math was boring or too hard. Through this survey, and after my literature review, it became obvious to me that I needed to change the way I planned my math units to start if I wanted to change the attitudes I was seeing in my math class.

The Unit Plan

I devised a unit on the topic of Ratios for my Grade 6 and 7 students (Appendix B). The unit employs some of the cognitive tools of the Romantic and Mythic understandings, as well as 3-Act Tasks, use of manipulatives, open-ended questions, and rich tasks of my own creation. This plan was revised several times throughout my research as a response to some of my observations and reflections.

On January 29th, I introduced the unit with a 3-Act Task. The video showed me placing two yellow balloons in a bag labelled For Division 11 and three blue balloons in a bag labelled For Division 12. After showing a few rounds of me placing two yellow and three balloons in their respective bags, students were asked to record their observations and questions and eventually they shared these out. As a class, we decided on a question to work on: How many balloons are in each bag? I randomly grouped students into groups of three and four, gave each group a whiteboard pen and assigned a space on the whiteboards, and the students were told they needed to work together to try to answer the question.

At the end of this lesson, I introduced them to the Heroic Quality of using Ratios: Finding equivalent ratios and being able to compare ratios can help us argue when something is unfair or unjust, and this skill can help us avoid getting ripped off. I gave several examples of how this could
be used in their lives (in Appendix B). At the beginning of each lesson, I would remind the students that the purpose of these activities was to practice how to make good, fair judgements about situations involving numbers. I also had a bulletin board titled “Ratios” with a list of the purposes of learning ratios (decide what is fair, get the better deal, avoid getting ripped off).

On February 5th, we revisited the Balloon 3-Act Task, and I showed pictures of the various solutions and strategies that students had used to try to determine how many balloons there could be in each bag. From there, I provided a note-taking page (Appendix C) that covered four different strategies that could be used to solve equivalent ratios, and we used manipulatives to create arrays.

The day after the note-taking, we used Skittles to work with ratios. In partners, students worked with Skittles to find ratios, equivalent ratios, and ratios in lowest terms. Students were not allowed to eat the Skittles until they had their worksheet (Appendix D) checked by a teacher.

On February 8th, after the Skittles activity, I had my teaching partner, Andrea, lead a conversation with both classes. I wanted to see if they were able to apply what they had learned in this activity. I also wanted to know if they felt a sense of play with this task. At the end of this conversation, students looked at three examples of student work (created by me), two of which
contained common errors in finding equivalent ratios, and one of which was correct. Students were asked to work together to determine the one that was correct, and to share their decisions.

On February 8th and 11th, we did an activity called “You Be the Judge”, which was a set of ratio stations that I created. I introduced the activity by wearing my father-in-law’s Supreme Court robe, and a white judge’s wig. I used a wooden mallet to establish “order” in the class and explained to them that they would have the opportunity to be a judge at each of the stations. The students were randomly grouped into four, given a package (Appendix E) and told that they needed to work together to make justified judgement of each scenario.

**Ratio Station Activities:**

*Station 1: Too Square or Not Too Square*

After seeing that a square has a side length ratio of 1:1, each group member creates different sized rectangles. They then work together to find the ratio of the side lengths and determine which person’s rectangle is closest to the ratio of 1:1.

*Station 2: Blue and Blu-er*

2 jars of blue solution contain different ratios of food colouring to water. Students work together, using equivalent ratios, to determine which solution is bluer.

*Station 3: Lemonade Stand-Off*

Students are given a scenario where they and their neighbour are in a competition to make the most money from a lemonade stand. The vendor with the sweetest lemonade will attract the most customers. Students make each recipe and use both taste and equivalent ratios to determine the sweetest lemonade recipe.

*Station 4: Graphs Can Help Us Too!*

Students look at a graph of a liquid to flour ratio in a muffin recipe. They then go to the following website [https://www.foodnetwork.com/shows/bakers-vs-fakers/11-baking-ratios-every-pro-should-memorize](https://www.foodnetwork.com/shows/bakers-vs-fakers/11-baking-ratios-every-pro-should-memorize) to find another ratio of liquid to flour, create a graph of this and compare it to the given graph.
Station 5: Nana’s Chocolate Milk

Students watch this 3-Act task (https://vimeo.com/37527166) and try to figure out how they can fix the mistake the person made when making Nana’s chocolate milk recipe.

Station 6: Wind-Up Toys

Students use a wind-up toy, measuring tape, and timer to determine the speed of the toy. They then compare this to the speed of my toy (given on the sheet) to determine whose toy was faster.

Station 7: Fair Payment

Students are given the following scenario: 3 children help with planting flowers. Charlie planted 9 plants, Rosie planted 18 plants, and Annaliesa planted 27 plants. Students need to determine what distribution of 66 marshmallows would be fair payment based on the number of plants each student planted.

On February 13th, we had a balloon propping race between the two classes, using the balloons that were distributed in the initial activity. My class had 22 balloons to pop, while the other class had 33 balloons. Students blew up the balloons, taped them to a table, and one student from each class was selected to pop the balloons, while two other students timed how long it took to pop them all. Students then needed to determine who was the fastest, using equivalent ratios and unit rates.

On February 19th, students were given an assessment where they had to come up with a real-life scenario that required the use of equivalent ratios to solve. Students were told that they would be creating their own question for this assessment the week prior. A copy of this assessment can be found in the appendices (Appendix F).

Following the ratios portion of this unit, I introduced the idea of ratio as percent through a 3-Act Task called “Calvin’s Clearance” (Pearce, 2016). After this task, students worked with some percent activities around the school (Appendix G). We also discussed tax, practiced how to calculate
it, and this culminated in a Shopping Spree Numeracy Task (Liljedahl, 2010) where students had to determine the costs (including tax) of items they would want to purchase.

Data Collection Methods

OBSERVATIONS AND ANECDOTES

On the days that I used 3-Act Tasks, the Skittles activity, and the You Be the Judge activity I collected data through observations as well as photos of student work. I was looking for evidence that small group activities promoted engagement in more learners than just the 11 who typically participated in the math conversations. I already had some evidence that showed me that large group, teacher-led conversations did not promote much engagement in the math conversations.

I took the research stance active participant (Zieman, 2012) and I used the technique that Zieman refers to as “jottings” to record what was happening in the classroom during these activities, conversations with between students, and I also used journaling at the end of each activity to reflect on the activity, make speculations as to what was going on, and to plan future activities. The challenge of being an active participant, as Zieman notes, is the juggling of both being teacher and observer. I recognized that my vested interest in this study put me in a position of anything but neutral while observing. To remedy this, I had used conversations with my teaching partner to get another perspective on the data I collected.

I also recognized that I held the assumption that these activities I had planned would promote a sense of play, relevance, and competence, and, that if they got the questions correct, this meant they truly understood the concepts. Therefore, I did not solely rely on these observations to make
my conclusions about engagement and understanding. I cross-referenced these observations with results of student assessments, comments in student reflections, and my teaching partners’ observations.

During the Classroom Conversation that took place on February 8th, I selected the research stance of the privileged active observer (Zieman, 2012). I sat in the back of the classroom, with a view of everyone, while my teaching partner led the discussion at the front of the class. I set-up my journal to observe the following:

- Number of hands raised during discussion
- Names of participants
- Participant responses
- Other observations

The conversation was led by my teaching partner, who had not been present for the Skittles activity. She elicited student responses about the activity and what they had learned. My aim was to see if more students contributed to this conversation than I had seen in the past, and to see if students felt a sense of play, a sense of relevance, and a sense of competence. I provided the following questions to my teaching partner ahead of the lesson and told her that I would be observing and making notes in the back.

1) What did you do on Wednesday?
2) What do you think we wanted you to learn from this activity?
3) What strategies did you use to answer the questions?
4) How did having the actual Skittles help you with this activity (and I don’t mean taste!)

STUDENT ASSESSMENT

At the end of the unit, I wanted to see if students could apply their knowledge of ratios to a real-world task. I had them create their own real-world problem that would require them to use equivalent ratios and/or proportional reasoning to answer their problem. I had students create their
own word problem, rather than give them one of my own, so that I could ascertain whether they could apply this concept to a real-world task. From this information, I could make some inferences about the sense of relevance. I also hoped to foster a sense of competence through this activity. By having students be the experts and keepers of the word problem (rather than the textbook or teacher), I would hopefully be able to have students see the value of their own knowledge. This assessment required students to use words, numerical operations, and pictures to explain their process. Their words would tell me if they understood the concepts on a practical level, the operations would tell me if they could use the concepts to solve their problem, and their pictures would tell me if they were able to transfer their play with manipulatives to this new situation.

STUDENT REFLECTIONS

At the end of the assessment, I had students reflect on the unit we had just finished. The reflection was prompted by the following three questions:

1) In a list or mind-map, write the activities we did throughout this unit on Ratios. List AS MANY as you can.
2) Do you think that understanding ratios is meaningful? Why or Why not?
3) Describe an “Aha moment”, a time that a concept in ratios started to make sense to you.

Through these questions, I hoped to see what they found to be most memorable (question 1), if they find the topic of ratios relevant (question 2), and what kinds of activities helped them gain a better understanding of the topic (question 3). I chose these questions to gauge students’ sense of play, sense of relevance, and sense of competence.

TEACHER CONVERSATIONS

Co-teaching lends itself to many reflective conversations with the person you teach with. In my circumstance, I had two co-teachers. Andrea, my main teaching partner, as well as Alice, who
teaches in Andrea’s class on Wednesdays and in my class on Thursdays. Throughout the unit, I noted some of the comments about the math tasks expressed by my partners, as this offered another perspective on the situations I was observing.

**Ethical Considerations**

Prior to beginning my fieldwork, our Education 904 class completed an ethics workshop, conducted by Dina Shafey from the SFU Office of Research Ethics. I informed my students and parents of my Inquiry project. I also obtained informed consent from the parents of 24 of my 25 students, and 20 of 26 students in the neighbouring class (Appendix G). My photos and examples of student work do not contain names or identifying features of the students. I informed my students prior to beginning my fieldwork that they while they would all be participating in the same activities, they had the option of asking for their work to not be considered in my study. At the time of writing this, no student has asked for their work to be disregarded from my study.

My teaching partners were a part of my research, and I obtained written permission to use their names and some of their observations in my report. I used member checking to ensure that my interpretations of their comments and our conversations were correctly accounted for.

**Findings and Analysis**

**RESEARCH DELIMITATIONS**

As my Action Research carried on, I made a few changes to the scope of my project to make the research manageable. Initially I had planned to conduct the study with both classes that I teach, however, as the sole researcher on this project, I quickly felt overwhelmed by the copious amounts of data I was sifting through. I decided to limit the physical collection of data (which also included
the marking of the data) to only my class. My analysis of the Initial Survey looked at the data from both classes, and I felt that it was necessary to use it all. The results of the Initial Survey strongly motivated me to change my approach to teaching for this unit, and if it were not for an overwhelming number of students telling me that they found math irrelevant, boring, or too difficult, I do not think I would have felt as inspired to change my ways. The observations and anecdotes I collected were of the whole group (all 51 students), while the data I collected on their assessment and reflections were from only the students in my own class.

ASSUMPTIONS

Throughout this study, I made several assumptions that allowed me to progress through my research. First, I assumed that all students would provide honest answers on their survey and reflections. When my students are speaking, I often hear them repeating things that I have said. However, when they write, they tend to share more honestly and individually. Perhaps this is due to the slowness of writing, causing them to rely more on their long-term memory than what might be in their working memory during a class conversation. Second, I assumed that students in Grade 6 had not yet learned about ratios, while the Grade 7’s had already had instruction on this in the previous year. I made this assumption because Grade 6 is the first year that Ratios appears in BC’s Revised Curriculum. Knowing how much content teachers feel they need to cover in math, it is unlikely that a Grade 5 teacher taught a Grade 6 concept to her students. Furthermore, Grade 6 is the first year of middle school, eliminating the chances of a Grade 5/6 split class.

TERMINOLOGY

Proficiency Language

Emerging – Initial understanding of the concepts and competencies in relation to the expected learning
Developing – Partial understanding of the concepts and competencies in relation to the expected learning

Proficient – Complete understanding of the concepts and competencies in relation to the expected learning

Extending – Sophisticated understanding of the concepts and competencies in relation to the expected learning

OBSERVATIONS AND ANECDOTES

Balloon 3-Act Task

Images taken from the video

As this was my first time using a 3-Act Task in my classroom, I found it challenging to be both teacher and observer at the same time. I am sure that I missed some observations that may be relevant to my findings.

During the video that showed me placing the balloons in each bag, students were almost completely silent (an unusual observation in math class). I repeated the 45 second video several times. By the third time I showed the video, all students were facing the projector screen, watching the video. This was noticeably faster than the typical time it takes to engage the attention of this group of students. In the past, it would often take close to 10 minutes to move all the students into one class, with their necessary materials, and have them focus on the board. My teaching partner, Alice, looked at me while I was replaying the video and whispered, “Wow”. I leaned in closer to her
and she said, “They’re really into this.” I could tell that this change of context in math class was effective in gaining students’ attentions quickly.

During the Notice and Wonder phase of the activity, students were mostly quiet – there were a few students whispering, but I could not hear what they were saying. My plan was to give them approximately five minutes to do this portion of the activity. At the end of five minutes, it was still silent, and many students were still writing in their notebooks. I took this to mean that they were still recording their Notice and Wonder. When I wandered around the room, many students were adding to their lists of observations.

When I asked the students to share out their ‘Notice’, I told them that anything and everything that they noticed should be shared. As they spoke, I wrote their ideas on the board. As students shared out their ideas, more and more hands went up. Here are some of the things they shared:

- “She’s putting balloons in a bag with our Divisions on it”
- “There are 2 different gift bags on the table”
- “She is wearing a green shirt”
- “One bag has flowers on it and the other bag has party stuff”

In total, there were 17 ideas shared out. When I got to the bottom of the whiteboard with their ideas, I moved on to the Wonder part. Several students who had still had their hands up let out groans. I found myself shocked that there were so many students willing to share their ideas: this was not typical in my previous math classes.

Many students raised their hand to share their Wonders as soon as I asked for their questions. I got the sense that after they saw me writing up every and all ideas from their Notice, more students felt comfortable with sharing their questions. Some questions they shared were:
• “Why is she putting balloons in the bag?”
• “What is the point of this?”
• “When was this recorded?”
• “What does this have to do with math?”
• “How many balloons are for Division 11?”

After we decided to answer the last question, students went to the whiteboards and windows to work through possible solutions. By the end of math, every single group had something written on their board space, and all groups students had attempted to answer the question.
Upon reflection of this activity, I noted that the number of students who participated by sharing ideas was much greater than previous math lessons. I see several possible reasons for this. First, there was no risk of being “wrong” by sharing out what one observed and wondered. Many students had noted in the initial survey that they do not participate in math conversations because they are unsure or are afraid of being wrong and this activity eliminated the ‘rightness’ or
‘wrongness’ and gave everyone an opportunity to engage with the task. Another possible reason for the increase in participation was the novelty of this kind of math activity. Student engagement could have been increased by the change of context in the math class. Other environmental factors that could have contributed to this was that my teaching partner was new to our class. (as my former partner is on a leave of absence). Perhaps they felt the need to impress her by appearing to be “on task” through this assignment. The lights were off during this activity. Perhaps dimming the lights could have had an impact on some students by removing some of the physical distractions from their view, or even promoting a sense of anonymity by darkening the room.

I was also surprised by the amount of writing that the students did to try to solve this problem (as shown in the photos above). I have often noted that students in my class do not show all their thinking in solving word problems from textbooks, and here I could see full thought processes written on the whiteboards. There are a few possible explanations for this. First, students were working in small groups. With 2-3 students coming up with possible answers, one would assume that there would be more ideas to write down than when students are working alone. Second, there have been studies that show that using vertical, non-permanent surfaces increase time spent on task, amount of writing, participation, and perseverance with math problem solving in the classroom (Liljedahl, 2018). In this case, every group was writing on a vertical, non-permanent surface. A third explanation could be that due to the open-ended nature of the task, students recognized that there were a few possible answers (as evidenced in pictures 2, 3, and 4).

Overall, I saw more participation, more math conversation, more perseverance, and more math writing than I had seen in any other math activity we had done this year. My intuition told me that the change of context was the main contributor to this: small groups, vertical non-permanent
surfaces, and open-ended tasks were a new concept for my students, and they bought in. My teaching partner, Alice, was so impressed with their engagement that she has been using 3-Act Tasks in the other classes she teaches. She has reported back to me that the engagement level of her students is high, and that the kids seem to enjoy this way of learning.

**Skittles Activity**

Students began this activity as soon as they received their Skittles. Photo 1 shows a group who had sorted their Skittles before I had a chance to give instructions. The incentive of being able to eat the Skittles after showing their teacher their work invited the participation of even my two students who often do not do much work in class (photo 3). Both students completed their Skittles sheet and handed it in, as did the rest of the participants.

For this activity, they had to work with a partner at their teacher-created table group. As I wandered around the room, I noted that all students were physically manipulating the Skittles, some of them even creating the arrays (photo 2) that we had done the day before.

Throughout this activity, students worked together not only in their partnerships, but across partnerships as well. They would help each other through the word problems, which were the more
challenging parts of the assignment. The excitement level was high in the room, yet students remained focused on the task throughout the activity. All students were able to finish the activity in the time frame given. There was an extension activity on the back, and more than half of the students attempted it – typically, in past activities this year, only a few of my high-achieving students would attempt an extension, often claiming that it was bonus, so they did not “need” to do it. Overall, I was impressed with the perseverance to complete the word problems and try the extension, as this is something that I had not seen with this group of students in the past.

Some comments that I heard from students were:

- “I can’t wait to eat the Skittles”
- “Why are we doing this with Skittles?”
- “Can we do this all the time with Skittles?”
- “This is torture!”

By the end of the activity, I felt that the activity was a success because of how each student was working to solve the problems. My teaching partner, Alice, even mentioned how engaged they were during the task. I felt almost guilty about bribing my students to learn math through candy. Upon closer look at the work they did on their sheet, I noticed a few common errors in calculating the equivalent ratios. It seemed that the previous day’s lesson had not yet crystallized. I also noticed that only about one-third of the students completed the word problems correctly. What I had thought to have been a successful, meaningful activity, now felt as though I had bought-off my students to participate. While they “did” the activity and practice of equivalent ratios, the extrinsic motivation of eating candy probably did not make this as meaningful. In fact, upon review of their work, I noticed several errors in simple calculations that had me question whether the Skittles were more of a distraction than a help.
During the following math lesson, my other teaching partner, Andrea, facilitated a class discussion about this topic. The following chart shows my observations.

**CLASS CONVERSATION OBSERVATIONS**

<table>
<thead>
<tr>
<th>Teacher Question</th>
<th>Student Responses (S)</th>
<th>Number of Hands Raised</th>
<th>Number of students engaged in side conversations</th>
</tr>
</thead>
<tbody>
<tr>
<td>What did you do in math on Wednesday?</td>
<td>NA</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>How many people remember Wednesday?</td>
<td>S: We got skittles, we got in partners, and we had to count them. We also had to do ratios and show our thinking for some. And word problems (the student proceeded to explain one of the word problems)</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>T: So the application of ratios</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do you think we wanted you to learn from this?</td>
<td>S₁: To feed us Skittles</td>
<td>4</td>
<td>~10</td>
</tr>
<tr>
<td></td>
<td>T: Add on</td>
<td></td>
<td>~25</td>
</tr>
<tr>
<td></td>
<td>S₁: Ratios</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>T: Add on</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S₁: Strategies</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T: Name them</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S₂: Equivalent ratios</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S₃: Comparing</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S₄: Lowest Terms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Who thinks they get these strategies</td>
<td>NA</td>
<td>~30</td>
<td>4</td>
</tr>
</tbody>
</table>
| How did it help having the actual Skittles for this? (And I don’t mean taste!) | **Student who rarely participates:**  
*Made me not want to work and eat*
*S2: It made me persevere*
*S3: some kids it’s easier to see it and easier to understand*
*T: Yes, some students are visual learners*
*S4: Kind of bad – I was rushing to get it done*
*S5: if this is going to help us do math, can we do it every time?*
*T: I don’t think Mrs. Murdoch’s bank account can handle this too often*
*S5: Moving around and seeing it helps*
| 6 (including a student who rarely participates) |

Reflecting on this I found three points of interest. First, the number of hands that were raised at the beginning of the conversation were not much different from the prior conversations in math. This once again had me consider that closed-ended questions in a large group setting run the risk of minimal participation. Second, several students identified that the Skittles were the reward, thus their focus was on finishing the math. Third, only two students identified that the Skittles also doubled as a manipulative.

Overall, this activity sparked participation, convinced students to do some math, and showed me the benefit of small-group activities. However, it may be an activity best suited for the end of a unit as a celebration, rather than activity to promote practice of concepts, as the focus seemed to be more on the eating of Skittles, and less on the practice of the previous day’s lesson.

**RATIO STATIONS**

I introduced this activity wearing a judge’s wig and robe. There was a noticeable “leaning in” as I talked to the students about being able to argue and defend themselves with numbers. The
students could see the stations set up around the class rooms, and this seemed to be a distraction for them, but in a good way. I could sense that they were eager to find out what we would be doing with the items I had laid out.

Students were randomly grouped in groups of four, given a package, and sent off to one of the 14 stations (there were seven different stations set up in each class). The students had a total of 55 minutes prior to lunch, and 30 minutes after lunch to complete the tasks. During this time most groups were able to complete three stations, while two groups completed 4 of the stations. The tasks at these stations required an application of multiple skills to compare the unlike ratios. I observed most groups playing with the manipulatives, making various calculations, and discussing solutions. Due to the complexity of the tasks, my teaching partner and I were wandering around, providing hints and tips to guide students in the right direction. Overall, students were highly engaged in the activities: not a single student was wandering around trying to find a distraction (something I had often observed during work time in my previous lessons). During this time, only two students asked to go to the washroom.

Two of the groups required teacher intervention to promote discussion between the members. In one of these groups, there was a student who quickly figured out how to solve the problems, but he did not discuss his methods with his other group members, who were struggling to figure out what to do. I sat with this group and asked the student to use the whiteboard to show and talk about his methods with his group. He was visibly uncomfortable with this but did his best to comply with my instructions. This was a group that I continued to visit, supporting all members to complete their task. The second group that needed intervention had very little discussion happening – they were all trying different methods on paper to try to solve the problem, but they were all using different strategies. As I continued to observe this group, I noticed that they were struggling with
finding an appropriate strategy. I sat with this group, provided scaffolding, and eventually they were able to find the solution to one of the stations.

When the stations were over, it was lunch time. Two groups continued to work on the station they were on. One of the students in the groups was a high-achieving student. I wondered if the rest of her group felt pressure to stay because of her determination to finish. When I approached the group and let them know it was lunch time, one of the students replied, “Can we finish this one?” I was tickled and took this as a sign that they were enjoying the station. As I was about to leave my room for lunch, a student from my teaching partner’s class approached me and asked if he could work on the Lemonade Stand-Off station after school. This was a student who is typically engaged in all math activities, but I was still impressed that he wanted to stay after school on a Friday to continue working on math. I said, “For sure!”

After lunch, my teaching partner suggested that we get the students to do one more station as they were clearly enjoying the activities. She shared a story of one of the students in her half of the room that was so determined to figure out the Nana’s Chocolate Milk that he stayed behind at lunch to get help with it. Again, this was a student who was often engaged during math lessons, but I had not yet seen students willingly give up their lunch break to work on math. We gave the students 30 minutes to complete one more station. During this time, I noticed the enthusiasm for these stations start to wane. One student, whom I later found out was not writing anything down in his package, ended up throwing the manipulatives at his station across the room, and breaking the ruler I had provided for the activity. I approached his group later and they said that he was difficult to work with and very distracted. Most of the groups were able to complete one more station, but there was a noticeable sigh of relief when we told them that we were moving on to another activity.
Throughout the stations, I noted two Grade 6 students who had what Peter Liljedahl (2005) refers to as an “Aha! Experiences” where the student reaches an understanding of a concept. Both students had typically struggled with understanding math concepts with my traditional teaching methods and often needed a lot of support with assignments. One of these students had his “Aha” moment when he was able to figure out the solution to Nana’s Chocolate Milk, and he was able to show one more of his group members how to solve it. He had arrived at the solution when I had drawn a visual of the problem on the board and blurted out “You add three cups!” I was very impressed with his determination as this student often had asked for help on every question of a math assignment. The thinking and perseverance I saw here was not typical of this student. The second student who had an “Aha” moment is one that I had sat with to explain lowest terms during the Blue or Bluer station. This was a concept that we had covered in the fall. She clearly had not understood the concept at the time, as I felt that I was teaching her something brand new during this activity. While I was explaining it to her, she exclaimed, “Ahhh! I get it!” and was able to find the solution to the problem.

At the end of the day, I asked my students what they had thought of the Ratio Stations, and here are a few of their responses:

- “It was fun – I liked being able to see and taste the examples”
- “Eh, I didn’t like doing it after lunch.” When I asked why, he replied, “I wanted to do my passion project”
- “We didn’t have enough time!”
- “This was more fun than a workbook”
- “Good”
- “Good because we got to work together. We got ideas from others.”
- “I think I would rather do a worksheet.” When I asked why, he replied, “It’s easier.”
- “Ehh…it’s math”
- “I don’t like sitting and listening in lessons, so this was good”
My analysis of the situation was that the stations themselves were engaging, and the small
group setting was mostly positive. I noted longer times spent on these activities than I had any other
math tasks we had done previous, even though the energy level started to dissipate after lunch. I
also noted much discussion about math strategies, whereas in my prior approach to teaching,
student discussion that happens during work time is usually not math related. I would like to think
that the title of the activity “You Be the Judge” helped them to see the heroic quality in the topic,
but nobody articulated this specifically in our end of day conversation. In hindsight, I would have
liked to have asked them if they had imagined themselves as a judge, trying to defend their
solutions. The one downfall to this activity was the time I spent setting it up. In total, it took me
close to 6 hours to set this up. However, now that it has been planned, it is something that I could
set up more quickly in the future.

**STUDENT ASSESSMENT**

The end of the Ratios unit meant that it was time for a student assessment. While I had seen
evidence of learning throughout the unit, I still think there is a need to determine what students can
do on their own, without the support of a small group. After all, I still need to give students
individual marks on their report cards. I call my math assessments MBY’s, which stands for Math by
Yourself. I told the students the week prior to this assessment that they would be creating their own
ratios question to solve on their MBY. I showed the students the following structure so that they
knew what they needed to be able to show us on the assessment:

- All students MUST be able to identify a real-world ratio (Emerging)
- Most students SHOULD be able to identify a problem that requires equivalent ratios
to solve (Developing)
- Some students COULD compare ratios with unlike terms in their problem (Proficient)
• Few students may use alternative strategies not yet explored to compare ratios with unlike terms (Extending)

My hopes were that students would go home over the weekend and plan out their problem. However, this was not the case. During the assessment, I was called over repeatedly by four students to help them come up with a problem. Of these four, two students were students that had always performed well on previous assessments. Perhaps due to the unfamiliar nature of the assessment, and their drive to do well, these two students felt that they needed to check in with me. My advice to them was to think about a problem that we had covered in throughout the unit and adjust the numbers and the items to make it their own. Even still, five out of my 25 students were not able to create a problem that made sense in words. I had to talk to them individually to get a better understanding of their problem. Once each student turned in their MBY, I asked my class, “How many of you went home and prepared a problem this past weekend to be ready for the MBY?” Only four students raised their hand.

I found myself disappointed that students had not taken advantage of the preparation they could have done for this assessment. However, when I took the assessments home and marked them, I was stunned at how well they had done with the task! It is important to note that for this assessment, I only looked at data from my class, and not the other class, as I was limited in time due to writing report cards simultaneous to this action research project. In the end, 12 students were able to come up with questions that had them find equivalent ratios, and 13 students were able to compare ratios with unlike terms. It has been unusual for me to have more students with the mark “Proficient” than the mark “Developing”. I recently double checked their assessment questions to make sure that I did not have the excitement of my action research project clouding my judgement,
and sure enough, I was still impressed with their self-created questions. The following pictures are samples of the questions they came up with.

Wanda makes 2 dozen cookies for selling on her street. One day, Emily's neighbor starts selling cookies too. She claims to have the most chocolate chips in her cookies than Wanda. Does she really?

Wanda's recipe: 5 cups of flour
3 cups of chocolate chips

Emily's recipe: 6 cups of flour
3 1/2 cups of chocolate chips

Great question! Who has the most chocolate chips?

One team won 31 another team played the same team and won 512. Who one by more?

A way to word this more accurately would be who had the "better win"?

Two friends Paige and Emma went to the mall. Paige gets pants at 11 and Emma gets shirts at 10. What is the ratio? Which deal is better?

Nice!
It is important to note that if the four students who had prepared for the MBY over the weekend had not done this, perhaps more students would have received “Developing” than “Proficient”. Ultimately, I wanted to know if students could think about the real-world applications of ratios, and I felt that they did.

While the wording of the student created questions was not always clear, I put on my ‘teacher as interpreter’ hat to make inferences about their intended meaning. I was most impressed that the situations and the numbers they used were more logical than I had noticed in my previous years of teaching. The 8.5 Litre pop bottle or sharing of 4 cookies in a class of 30 written by my previous Grade 8 students had me wondering if richer math tasks could have rectified this situation. I did have 3 students use some unusual numbers, as in the example below, however, two of the students were in grade 6, and I can only hope that over time, with more practice of real math scenarios, “real life” and “math” will become synonymous in their thinking.

Create a real-life word problem that involves ratios and using equivalent ratios. This situation should use different numbers and comparisons than what we have done in class.

Mason and James are walking to school. Mason walks 86 metres to school in 27 mins, James walks 21.5 metres to school in 13 mins. Who walks faster?

We have some slow walkers in this scenario!

From this final assessment, I could see that the unit and activities we did on ratios may have helped to create a sense of relevance with this math topic. However, my teaching partner, Andrea, commented that her students seemed to struggle with creating a question that needed to use equivalent ratios to solve. She noted that they were able to calculate equivalent ratios, but their
questions did not always make sense. I decided that I needed to look further into my students’ reflections to see if a sense of relevance was felt amongst them

**STUDENT REFLECTIONS**

Students were asked to complete 3 questions on the back of their MBY. 1) *In a list or mind-map, write the activities we did throughout this unit on RATIOS. List AS MANY as you can.* 2) *Do you think that understanding ratios is meaningful? Why or why not?* 3) *Write about at “Aha” moment.*

24 out of 25 of my students completed this reflection (one had been away on holidays when we did this and ran out of time during her rescheduled assessment). Once again, due to time constraints, I only looked at my class reflections.

In the first question, most students listed many of the activities we had done while learning ratios, such as Skittles Activity, comparing ratios, equivalent ratios, Balloon Popping Race, the various Ratio Stations tasks, and one person even wrote “worksheets on Thursdays” (the day I do not teach). The following reflection was done by a Grade 6 student:
I was most impressed with her category of “stating your case”, where she added “proving your point”. This struck me, as I took it to mean that my Heroic Quality of being a good judge was something that stuck with her. To verify my analysis, I asked her what made her write this. Her reply was, “It was the whole point of learning this, wasn’t it? So we could be a good judge?”. Yes! It was!

While only one student identified this as something we “did” this unit, it was enough for me to feel like using a Heroic Quality was effective in not only engaging but also in adding meaning to what we “do” in math. In hindsight, I wish I had included the following reflection question on the back of this MBY: Describe what it’s like to know that you can now be a better judge of what’s fair, what’s right, and what’s equal.

The second reflection question asked students to discuss the meaningfulness of this unit. I wanted to see if students could see the real-world applications of ratios, and I also wanted to know if they could find the relevance of ratios now. Out of the 24 responses, one student said that she did not see the point of this, because she already knew how to compare numbers. Seven students wrote that they could see this skill being useful either in a job or later in life. The remaining 16 students said that they could see that ratios could help them in their lives and gave examples such as baking, buying things, comparing situations, and one student even wrote that he could see this helping him win arguments with his brothers. Comparing these responses to their Initial Survey responses, where 89% of students said that they thought the purpose of math was for their future, only 29% of respondents identified ratios as a skill only good for future use. Unfortunately, due to the separation of our two classes for this reflection, I did not get to see the responses written by my teaching partner’s students. However, even if all 26 of her students had written that ratios were solely skills for future use, this would still be fewer than the number of students who indicated this
on the initial survey, leading me to believe that students were starting to see the relevance of learning ratios.

The last reflection question had students discuss an “Aha” moment. I was curious to see if they felt a sense of competence, and what situation allowed for this to happen. From their responses, only one student wrote that he did not have an “Aha” moment. He did not elaborate, and unfortunately, after I looked at this response, he was absent for 8 days prior to Spring Break. I would have like to have followed up with him on this. Of the 23 other students who did have “Aha” moments, 10 of them identified that it was when working with me on a concept, whereas the other 13 identified their group members who helped them through a concept. Upon this discovery, I immediately thought, “Wow, I certainly do not need to be running these lengthy lessons, where students are expected to take copious amounts of notes for learning to happen.”

Reflections on the Process

Due to my busy schedule as a student, a teacher, and a mom, I like to have a plan. I plan my meals, my teaching, my weekends – I often know what I will be doing. Throughout the research process my plans evolved and changed, almost daily. In the beginning I spent copious amounts of time planning activities for the unit on ratios, many of which I did not use in my action research. What I did not realize, was that much like any other plan in teaching, plans are destined to evolve and change as you engage in this kind of research. While I knew that change was inevitable, I felt that I needed to be as prepared as possible prior to starting my research. Had I taken the advice of the former researchers, I perhaps would have put more time into collecting more preliminary data on attitudes in math, rather than planning activities that were of no use.
Another approach I would take in future research, is to video record my students during the class conversations and tasks. My observations were limited to what I focused on at any given moment. Had I recorded the 15-minute discussion, or any other activities, I would have been able to see what was happening with all the students during this time. I do recognize that reviewing video footage would have been a time-consuming endeavour, and due to my limited time as both researcher and teacher of all subjects, I did not include this in my data collection methods; however, it would have been nice to verify some of my observations.

Conclusions and Implications

My original question was, “How can using rich math tasks effect student engagement?” As Fredericks notes, it is quite challenging to measure engagement, especially after only several interrupted weeks of research. However, after using rich math tasks, I observed and felt a change in my students’ engagement with math. I heard more math conversations, saw more time spent on tasks, and felt more of a positive energy from my students during these tasks. My teaching partners also observed these changes, and Alice is starting to use 3-Act Tasks and open questions in the other math classes she teaches. My ratios unit was premised on the heroic quality of “being a good judge” and this, I feel, was the crux of evoking a sense of relevance, fostering a sense of competence, and creating a sense of play with my students.

EVOKING A SENSE OF RELEVANCE

Prior to starting my action research, many students saw that the purpose of math was for future use only. By the end of my unit, fewer students identified ratios as something only necessary
for their future, and rather, many students saw ratios as a skill to help them determine fairness and better deals. A sense of relevance was evoked when I showed the unfair distribution of balloons in the two different gift bags. In math, students often ask the questions of ‘why?’ ‘What’s the point?’ Or the age-old question, ‘When are we ever going to use this?’ This time, students were able to discover the answer to these questions for themselves. Pitting the classes against each other in an unfair balloon popping race convinced the students to use equivalent ratios to determine who the fastest was. (In case you’re wondering, Division 12, who had more balloons, was the fastest at balloon popping). This sense of relevance continued when we started looking at percent discounts and finding the better deal – the heroic quality of “not getting ripped off” continued to bear significance and relevance.

**CREATING A SENSE OF PLAY**

Throughout my research, students were using manipulatives and visuals, they were collaborating, and they were “playing” with their ideas. As Marian Small suggests, open questions promote this kind of play that is necessary for learning (2010). During the math tasks, I felt more enjoyment from my students – I did not observe as many bathroom visits, water breaks, or off-topic conversations. I had students who gave up their lunch break to work on tasks, I saw more time spent focused on tasks, more conversations about the math problems, and I even had a student who wanted to stay after school on a Friday to complete a math task. My traditional teaching with a lesson and textbook questions did not have this same effect.
FOSTERING A SENSE OF COMPETENCE

All students were able to work through the 3-Act Tasks without my intervention, without me first teaching a concept or skill, and without the internet to help them. Based on my upbringing, it seemed rather counterintuitive that students were better able to solve a math task without first learning the strategy or algorithm but giving them the freedom to try their own strategies proved to be beneficial. While a sense of competence is difficult to measure, I was able to observe an increase in time spent on tasks, and not a single group ever ‘gave up’ or said “I don’t get it” like I had observed in math prior to this action research. I interpreted this to mean that students felt that they had the capacity to solve the given problems. As Boaler (2016) notes, open-ended tasks (such as 3-Act Tasks) allow all students to engage with the task without the idea that there is a “right way” to solve it. By providing open-ended tasks, all students, even my two students who often do not do work in class, were able to contribute something to the math conversations that led to the solving of a problem. An area for future research could look at how starting off a math unit by teaching algorithms and strategies may limit student ‘play’ with alternative strategies.

SO, WHAT?

In the short amount of time that I tried open tasks with my students, I noticed an overall positive change in engagement with math. When I think back to how I used to teach math (stand-and-deliver lessons, note-taking, and textbook questions), I can understand why there was a lack of engagement and logical thinking in my math class. And yet, I was following a program that all our math textbooks outline. Our math textbooks and teacher guides begin with a lesson that teaches an algorithm, has the students practice the steps in the algorithm, then provides questions and word problems to practice the algorithm. This prescribed process of teaching math that our textbooks
follow does not lend itself to promoting a sense of play, competence, or relevance, which are all important elements in students learning math.

Textbooks are valuable resources for both students and teachers, and in no way am I suggesting that they are not. I would suggest that teachers start their math units with open tasks, such as a 3-Act Task. I recommend that they allow students to take their time with these tasks and come up with strategies and solutions of their own. This process alone promotes a sense of play, a sense of competence, and if the task is chosen wisely, it can promote a sense of relevance. Then, after time has been spent fostering these feelings of play, competence, and relevance, the textbook could be used as one resource for practice of the concepts.

**NOW WHAT?**

No action research would be complete without a feeling that I have more questions than answers. While I can clearly see the benefits of using rich math tasks in math, I am left wondering if there are any benefits to my traditional way of teaching math (to not ‘throw the baby out with the bathwater’). First, is there any value in taking notes in math class? Perhaps after debriefing a 3-Act Task, students would then go around to the different whiteboards to write down the different approaches that various groups tried. Second, do rich math tasks inherently promote a deeper understanding of concepts – or is it just the change of context? In other words, would the novelty of 3-Act Tasks wear off after a while? A longer study would be required to answer this. Third, does this approach to teaching math have the same effects among different age groups? Would I notice an increase in engagement in Grade 12? Or Kindergarten? Fourth, as our province is still figuring out a numeracy assessment for our high school students, could 3-Act Tasks be used as an assessment tool? As I am only 15 years into my teaching career, I look forward to continuing this approach to
teaching math and contributing to the conversations around the math resources and assessment practices we use in the coming years.
Works Cited


Appendix A – Initial Survey

Thank you for sharing your thoughts with me on this survey. I would like for you to take your time, be detailed about your answers, and above all, be honest. I will be using this information to help me become a better teacher 😊.

1) Why do you think we learn math at school?

2) Is math important? Why or why not?

3) What are the characteristics of a good mathematician? List as many as you can think of.

4) From the characteristics you listed above, which ones do you have, and/or which ones would you like to work on?

5) What do you do when you come to a math problem that is challenging? Choose as many that apply to you. Be honest 😊.

- Try some different operations (add, subtract, multiply, divide) to see if any of them make sense.
- Go to the bathroom.
- Draw a picture of what the problem is asking.
- Ask the teacher.
- Go get a drink of water.
- Talk to your friends.
- Draw a picture.
- Read over your math notes to get ideas.
- Ask a friend for help.
- Use the internet to find solutions.
- Skip it.
- Ask your parents for help.
- Other ______________________________

6) Describe math lesson or activity that you found enjoyable or were proud of over the past 5 years. What was the topic you were learning?

7) Do you participate in the math conversations we have during lessons we do in class? It’s ok if you don’t 😊. If you do not usually participate, I would like to know why.

Possible reasons:
• I am unsure if I have the right answer
• I am shy
• My teachers have never asked me to
• I find it hard to concentrate where I am sitting
• I am thinking about other things
• I am not sure what to do
• Other _____________________

8) Tell me about a time you used math outside of school.

9) Do adults (other than your teachers) ever talk about math ideas or concepts with you? Explain.

10) Is this year’s math challenging enough for you? Explain.

11) Do you think most people enjoy math? Why do you think this?

12) What do you hear others say about math?

13) Create a mind map of all the things we have learned in math this year. Add lines and words that show the connections between the things we have done.

Possible vocabulary words to help you start:

Place Value  Multiple  Factor  Fraction  Decimal  Percent

Graphing  Data Analysis  Addition  Subtraction  Multiplication  Division
Appendix B – Romantic Ratios Unit Plan

Planning Framework for Middle/Secondary School Teachers: Employing the Cognitive Tools of Romantic Understanding

1. Identifying “heroic” qualities

Understanding how to use ratios to compare rates will help you become good at arguing for what is fair, just, and rightfully yours.

2. Shaping the lesson or unit

2.1. Finding the story or narrative:

Imagine coming downstairs on Christmas morning, beating your younger or older sibling to the beautifully lit tree, and noticing all of the presents that Santa had brought the night before. Impatient for your family to wake up, you start rifling through and finding all the presents with your name on it. As you are sorting your presents, you notice that your sibling has 10 gifts and you have 7. You wonder, is this fair? Is the value of your gifts equivalent to the value of your sibling’s gifts?

OR

Imagine that you are purchasing pop for your class party. You go to the store with the $5 your mom gave you, and you see that there are 2 options: a 2 litre bottle of Coke for $2.99 or a 6-pack of cans of coke for $3.99. Which is the better deal?

2.2. Finding extremes and limits:

There are many bizarre records of speed noted in the Guinness Book of World Records. One such example is:

Balloon Popping Race: The Guinness Book of World Records holds a record for the dog who can pop balloons the fastest. Video link:

https://www.youtube.com/watch?v=X-_tqd7e_D8

Looking at various speeds in the Guinness Book of World Records is a great way to start calculating unit rates.

2.3. Finding connections to human hopes, fears, and passions:

If you do not know how to determine what is fair and just, you might get ripped off.

Humanizing the context by having students figure out the strategies to solve (rather than depending on a textbook formula or teacher-led lesson) will also give them the sense that they can figure out math tasks with the skills and knowledge they already have.

2.4. Employing additional cognitive tools of Romantic understanding:

- **Collections and hobbies**: There are several ways to calculate rates and equivalent ratios. My hope is that through 3-Act tasks, we will be able to make a collection of the variety of methods that can be used (e.g.: charts, manipulatives, calculations, graphs, visual drawings)

- **Change of context**: Rather than the teacher leading the lessons on “how to” with equivalent ratios, students will be shown a video of a real-life scenario and they must come up with ways to solve this. The students will become the experts and keepers of the knowledge.

- **The literate eye**:

Using charts, arrays, and graphs can help us calculate and predict equivalent ratios. Students will need lots of practice with these tools to be able to use them effectively. Individual whiteboards and coloured tiles are great ways to teach these.

- **The sense of wonder**: 

First 3-Act Task – with a video showing an unfair distribution of balloons between the two classes, students will be wondering why it is set up this way (this gets resolved in the conclusion)

You be the Judge: Students will be tasked with trying to find the best, the sweetest, the bluest, the fairest in the various ratio stations.

Balloon Popping Race: Students will be curious to find out who the winner of the Balloon Popping Race is when the number of balloons is not equal.

3-Act Tasks: The first videos in these activities ask students to notice and wonder. After students share their wonderings, we will attempt to answer both the mathematical questions, as well as the other questions they generate.

- Embryonic tools of philosophic understanding:

  While ratios and rates are great ways to determine fairness, equality, and getting what you deserve, numbers are not the only factor that decide this.

  Finding anomalies to the general theory will come up when we discuss the ratio stations. For example, with the Lemonade Stand-Off activity, students may present (or teacher can present) the idea that perhaps the less sweet lemonade would appeal to more adults than the extra sweet one.

  Or in the activity that involves determining a fair wage, we might note that the person who planted the fewest amount of plants could have had more challenging tasks to complete than the person who planted the most amount of plants.

2.5. Drawing on tools of previous kinds of understanding:

  Somatic understanding

  Many of the activities planned for this unit food and hands-on materials. The Skittles Activity, the Ratio Stations, and the Balloon Popping activity are fun activities where taste, touch, and sound are stimulated.

  Mythic understanding

  Binary Opposites

  Fair vs Unfair

  Just vs Unjust

  Games: The Balloon Popping activity will evoke a sense of competition, and of course, they will need to figure out who was the fastest balloon popper using unit rates.

3. Resources

3-Act Tasks (Dan Meyer, Kyle Pearce, John Orr)

Good Questions (Marian Small)

Building Proportional Reasoning (Marian Small)

4. Conclusion

The Balloon Popping game will be the final activity (prior to assessment) that has them determine a strategy for figuring out who was the fastest balloon popper. This brings the first 3-Act Task (balloon distribution) to a close, and students will be able to see that fairness is more than just a set of numbers.

5. Evaluation

Our final assessment at the end of the unit will involve students coming up with their own scenario that involves using unit rates and equivalent ratios to solve.
Appendix C – Ratio Note-Taking Page

Ratios

Ratios are

______________________________________________________________________

YOUR TURN

Think back to the Gift Bags of Balloons: What were we comparing?

______________________________________________________________________

We can write them in 3 different ways:

<table>
<thead>
<tr>
<th>Words</th>
<th>Numbers</th>
<th><em>Fractions</em></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

These are called _______ to _______ ratios because they are comparing 1 part of the balloons to another part of the balloons.

YOUR TURN

Ratio of Div. 11 Balloons to Div. 12 Balloons _______________

Comparison of Div. 12 Balloons to Div. 11 Balloons _______________

If we wanted to compare the number of balloons in Div.11 to the TOTAL NUMBER of balloons, that would be called a__________ to __________ ratio, because it compares one part to the WHOLE amount.

YOUR TURN

Comparison of Div. 11 Balloons to total balloons _______________

Comparison of Div. 12 Balloons to total balloons_____________

Equivalent Ratios

YOUR TURN

Each time Mrs. Murdoch put the balloons in the bag, I did it at a ratio of 2:3 or 3:2. What were some of the possible numbers that were in each bag?
STRATEGIES YOU CAN USE

Make a Chart:

Draw a Picture:

Numerically:

Graph it:

*Fractions*

The fractions of ratios are used to compare amounts.

YOUR TURN

Use the following words and phrases to make a true sentence about Division 11 balloons and Division 12 balloons.

\[
\text{Div. 11} \quad \text{Div. 12} \quad \text{Two-thirds} \quad \text{as many as}
\]

Extension: write the fractional sentence that compares Div. 12 balloons to Div. 11 balloons.
Appendix D – Skittles Activity Sheet

Skittles Ratios

Yay! SKITTLES!! Before you eat ANY, you must show this completed page to one of your teachers 😊.

Fill in the table.

<table>
<thead>
<tr>
<th>Colour</th>
<th>Number in bag</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td></td>
</tr>
<tr>
<td>Red</td>
<td></td>
</tr>
<tr>
<td>Orange</td>
<td></td>
</tr>
<tr>
<td>Yellow</td>
<td></td>
</tr>
<tr>
<td>Purple</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
</tr>
</tbody>
</table>

1. What is the ratio of orange Skittles to the total number of Skittles? Extension: write this in lowest terms.

2. What is the ratio of green Skittles to red Skittles? Extension: write this in lowest terms.

3. What is the ratio of purple Skittles to red Skittles? Extension: write this in lowest terms.

4. Write the number of orange to yellow to purple skittles as a ratio.

   Orange : Yellow : Purple = .................................................................

5. If you had three containers of Skittles, exactly the same as the container you were given, how many Skittles would you have all together? Show your thinking.
5.a What would the ratio of purple Skittles to red Skittles be? *Use one of the strategies for finding equivalent ratios

Strategy I will use __________________________

5b. What would the ratio of green Skittles to red Skittles be? Use a different strategy to find this. Choose a different strategy than you did for #6.

Strategy I will use______________________________

6. Compare your answers for question 2 and question 5b. What do you notice? Use “mathanese” to answer this.

7. There is a bag with 30 Skittles inside. Ms. Murdoch, Ms. Coupe and Ms. Moore share the Skittles in the ratio 3 : 2 : 1. How many Skittles do we get each? Try an AREA MODEL!
   Extension: There is a bag with 72 Skittles inside – solve.

8. Ms. Moody likes to mix her Skittles when she eats them. She insists that 4 cherry with 2 grape is the best flavor combination. How could you create this same flavor combination with the same level of cherryness and grapeness if you only had 3 grape and 2 cherry? Show and tell your thinking, but don’t eat any…yet!

Extension

9. Ms. Murdoch and Ms. Moore also like combining the flavours of Skittles. Ms. Murdoch says that a ratio of 2:3 yellow to orange Skittles tastes lemony. Ms. Moore argues that a ratio of 3:5, yellow to orange Skittles tastes lemonier. How could you use math to prove who is correct (and NO, you can’t just taste-test 😊😊).
You Be the Judge

You and your group members will travel from station to station, working together, figuring out the BEST, the BRIGTHEST, the FAIREST.

A Good Lawyer (and Judge) always:

- Presents evidence (in math, this means SHOW YOUR REASONING)
- Has a closing statement (in math, this means write a sentence that answers the questions!)
1. Too Square or Not Too Square

(Which rectangle is CLOSEST to square?)

The lengths of a side of a rectangle that has a ratio of 1:1 is called a SQUARE (which could be 1cm x 1cm or 2cm x 2cm, or 3cm by 3cm etc...)

Draw 2 different sized squares here. Or, draw it here (but use a ruler!!)

Now, draw a rectangle that’s CLOSE to a square, without it being a 1:1 ratio. Write the ratio of the side lengths. Use the grid IF you want, or draw in the blank space beside.

Choose someone with a DIFFERENT ratio of side lengths: ________________________(person’s name)

Ratio of partner’s rectangle side lengths: ___________________

Now, decide whose rectangle was CLOSEST to square. Be sure to communicate ALL of your thinking and Math!!
2. **Blue and Blue-er**

*(Which is MORE BLUE?)*

Solution A contains 3 parts blue food colouring to 50 parts water. This can be written as a ratio of ________________.

Solution B contains 5 parts blue food colouring to 75 parts water. This can also be written as a ratio of ________________.

Which solution is MORE blue?

Explain how you know with numbers and words!

3. **Lemonade Stand-off**

*(Which is the SWEETEST?)*

Imagine you have been running a successful lemonade stand. Every summer, you make loads of money, selling your special recipe of lemonade. BUT this summer, your neighbor decides to offer a little friendly competition and SHE opens a lemonade stand, claiming to have the SWEETEST lemonade.

**YOU BE THE JUDGE. Make the two lemonade recipes in the jars provided.**

**YOUR LEMONADE RECIPE:**

- 15mL lemonade powder (Be sure to level this with the edge of the knife), 120mL water.
- STIR, STIR, STIR!
YOUR NEIGHBOUR’S LEMONADE RECIPE:

- 10mL lemonade powder, 100mL water
- STIR, STIR, STIR!

In order to make the most money, you will not be giving out taste tests to your customers. How can you prove to your customers, with MATH, that yours is the SWEETEST?*Be sure to wash out the jars for the next group!

4. Graphs can help us too!

(1:1 is equal...what does CLOSE to EQUAL look like?)

Look at the graph on the wall. What does it show? Describe the ratio this shows.

Look up another recipe that has a different ratio of flour to liquid (Google – 11 Baking Recipes Everyone Should Know) and graph a ratio of Flour to Liquid. Be sure to title your graph with the Ratio you find.

Chart

<table>
<thead>
<tr>
<th>Flour</th>
<th>Liquid</th>
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Graph

What do you notice about the angle of your line compared to the angle of the Muffin Recipe?
How could a graph help you to decide what is EQUAL?

5. Nana’s Chocolate Milk

(How do you make it JUST AS GOOD!)

Watch the video. (or google Nana’s Chocolate Milk Act 1 and watch this!)

How can we fix the cup of chocolate milk to Nana’s favourite? If you are adding or taking something away, be sure to show HOW MUCH and prove that you are correct!

6. Wind-up toys!

(Whose toy is FASTER?)

Mrs. Murdoch’s wind-up toy travels 35 centimeters in 7 seconds, you would write this as a speed of 35cm/7s. This is an example of a RATE (kind of like the word ratio, huh?)

Distance Traveled: __________________________ cm

Time it took: _______________________ sec

Rate: ____________

A UNIT RATE tells you how far it travelled in 1 second. (UNIT = UNicycle = UNiverse = 1...Get it???)

Use the line to figure out how far it travelled in 1 second.

Now for some FUN!

• You will need to measure the speed of the wind up toy.

Instructions

1) Get a timer and measuring tape.
2) Wind up the toy, being careful to not overwind it.
3) Place the toy on the start line, and let go as you start the timer.
4) Stop the timer when the toy stops.
5) Record time ___________________ seconds
6) Now, measure the distance it travelled with the tape measure ______________________ cm
   (you may need to wind the tape measure around the path it took!)
7) Write the speed or rate that it travelled ________cm/_________s.
8) Can you figure out if this one is faster than mine?

7. Fair Payment
(How should the payment be distributed?)

Mrs. Coupe hired 3 students to plant plants in her garden, and she promised to pay them in
marshmallows. Charlie planted 9 plants, Rosie planted 18 plants, and Annaliesa planted 27 plants.

Write a three-term ratio to describe this relationship.

Can you reduce this to lowest terms?

Mrs. Coupe gave them 66 marshmallows to split between them. What would a fair distribution of
marshmallows be. Use the marshmallows and the chart to divide these.

Draw an Array and write a Sentence that describes the fair payment.
Appendix F – Ratios Assessment

RATIOS MBY – Show us What You Have Learned

Create a real-life word problem that involves ratios and using equivalent ratios. This situation should use different numbers and comparisons than what we have done in class.

Use numbers to show calculations for the situation you wrote above.

Draw a visual representation of your scenario.

Write a sentence answer that clearly communicates an answer or solution to your problem.

Reflection
In a list or mind-map, write the activities we did throughout this unit on RATIOS. List AS MANY as you can.

Do you think that understanding ratios is meaningful? Why or why not?

Write about an “AHA” moment – the times that a concept in ratios started to make sense for you and you felt you had a better understanding. Consider the following questions in your response.

- What was the activity you were doing?
- What was said or shown to you that helped you to understand the concept better? Who helped with this?
- Describe your new understanding of the math concept?
- How did you feel about coming to this AHA moment?
Appendix G – Informed Consent

Dear Parents,

I am currently working on my Master of Education in Imaginative Education through Simon Fraser University. This program enables me as an educator to reflect upon my practice and its impact on my teaching, as well as on my students’ learning, with the intention of developing my own best practices. As part of my studies I have developed an inquiry project to examine how I can enhance my math teaching through engaging student imagination. I anticipate that my inquiry will provide me with insights that will help me create more engaging and meaningful math lessons, as well as enhance my teaching in other areas.

My inquiry will be primarily informed based on my own observations and reflections on my work as a teacher. Over the course of the next two months I will also collect student work samples, surveys, and reflections, to inform my understanding of my practice. All elements of my inquiry will take place within the context of my normal instruction and practice.

This letter of informed consent is part of my ethical responsibilities as a teacher-inquirer. I am asking your permission to use your child’s work samples, surveys, and reflections to present to members of my graduate cohort and my instructors to demonstrate my own learning. As part of my responsibility as an educator, professionalism around issues of confidentiality will be ensured. Consistent with the ethical protocols of teacher inquiry, if your child is mentioned in the presentation of my work, an alias (pseudonym) will be used at all times to respect and protect his/her privacy. I would like to reassure you that regardless of my inquiry, my ethical best practices as a teacher will remain the same.

This inquiry process is not intended to assess, place, or evaluate your child in any way, but will serve to strengthen my teaching practice. Regardless of your decision, the integrity of the relationship I have with your child will not be affected, and you can withdraw your consent at any time.

If you have any questions or concerns, please don’t hesitate to contact me at dmurdoch@sd43.bc.ca. If you agree to give your permission, please sign below and return at your earliest convenience.

Thank you for your consideration in this matter.

Sincerely,

Danielle Murdoch
Team Wolf – Division 11
Eagle Mountain Middle School

Child’s full name: _________________________________

I, ______________________ give permission for my child to be included in Danielle Murdoch’s inquiry and for the collection of work samples, surveys, and reflections.

Parent Signature ________________________________ Date________________________